The Teacher as a Broker in Establishing a Classroom Community of Practice

by Phil Clark

Arizona State University

Abstract

The purpose of this research is to describe the role of a teacher as a broker in a *classroom community of practice*. It considers the teacher's actions as they occur in the classroom and how they effect students' mathematical activity. The term broker is used to recognize that the teacher was a member of the classroom community and the larger community of mathematicians. This segment of a semester long study builds on the notion of community of practice to frame the social setting of the classroom and looks at student activity within this setting.

Introduction

While the role of proof in the mathematics classroom has some discrepancies amongst the mathematics community, there is agreement that learning to prove is a necessary skill in mathematics. Often, students who decide to take upper level mathematics courses based on their success and enjoyment of calculus have difficulties with proof oriented classes and come to dislike mathematics (Tall, 1992). Most research has shown that many students are not able to create the deductive arguments necessary to write valid proofs (Moore, 1994; Selden & Selden, 1995, 2003; Dreyfus, 1999).

Recently there has been a lot of literature in Mathematics Education reporting that classrooms in which communities of practice emerge foster learning and an increased affinity for mathematical activity (Schoenfeld, 1992; Steffe & Gale, 1995; Wenger, 1999; Cobb, McClain, de Silva Lamburg, & Dean, 2003; Cobb & Hodge, in press). Many of these studies highlight the importance of social and cultural processes when accounting for students' mathematical growth. Although research has been done concerning students involved in an introductory proof course, little of it has focused on the social setting of the classroom. Instead most of the studies were concerned with the way students were individually attempting to write proofs and the difficulties they encountered (Finlow-Bates, Lerman & Morgan, 1993; Moore, 1994; Weber, 2001).

Since proving can be seen as a social construct (Yackel, 2001), some researchers suggest that learning to prove should take place in a community where students are encouraged to share their thoughts and reasoning (Alibert & Thomas, 1991; Cobb & Yackel, 1996). Therefore a model of a class in which a community of practice emerges may aid in this transition and could help inform future instructional design. The teacher, acting as the representative of the mathematical community, can create an atmosphere which encourages these types of normative behaviors (Cobb, 2000).

The purpose of this research is to document how students taking an introductory proof class in collaboration with the teacher formed a classroom community of practice. The term classroom community of practice is used to denote that while a classroom shares many traits with a community of practice as defined by Wenger (1998, 1999), it is actually a smaller, transient version of an actual community of practice. It is within these classroom communities that the participants can create the meaning for themselves and make sense of the mathematics as it is constituted in the classroom (Cobb & Hodge, in press). During this study I plan to give evidence of how a mathematical structures class was becoming a classroom community of practice in the beginning of the semester and how the teacher fostered its emergence.

Framework

A community of practice emerges when a group of individuals engage in the pursuit of a joint enterprise that encompasses communal interests (Lave & Wenger, 1991, Wenger 1999). A joint enterprise encompasses of the goals and values of the members of the community. These normative, communal interests are jointly established by the classroom community and are continually regenerated by the teacher and the students in the course of their ongoing classroom interactions (Cobb et al., 2003).

According to Wenger (1998) brokers provide a bridge between the activities of different communities by facilitating the translation, coordination, and alignment of perspectives and meanings. Cobb, McClain, de Silva Lamburg and Dean (2003) identified the use of brokers as necessary in helping to align the goals of three distinct communities of practice. They found that when they added brokers to help coordinate the perspectives of the administration, the math content experts and the math teachers, it made it possible to accomplish the goals of each community.

A classroom community of practice emerges when the students, along with the teacher, progressively work together and learn to adhere to the epistemological values held by mathematicians and communicate by the conventions of the larger mathematical community (Lave & Wenger, 1991). It is within this classroom community of practice that the teacher acts as a broker to help guide the students to engage in activities in a way that is commensurate with the larger mathematical community. Thus the teacher can be seen as a broker bridging the gap between the two communities and aligning the goals of mathematics classroom with those of mathematical community. They encourage this alignment by working to establish the particular norms of the classroom in collaboration with the students, by introducing activities, and by introducing formal conventions accepted by the larger mathematical community.

Methods

This data was collected during the first several weeks of a semester long teaching experiment (Cobb, 2000) in a mathematical structures course where students' transition from the computational mathematics of calculus to the more rigorous proof writing required for upper level mathematics. The students engaged in activities that encouraged them to use formal deductive arguments to justify their conclusions. The students worked in small groups (about 3 to 4 people) and often had whole class discussions regarding the activities. The classroom was videotaped using two cameras. Data came from the transcripts of the videos, written work, reflective emails and field notes taken by the researcher during the semester. The transcripts were coded for areas in which the students and teacher were observed contributing to the norms of the classroom and points where the teacher's influence helped student learning. These instances were inferred by identifying regularities in patterns of the students' social interactions.

Results

The following results are from the first two weeks of class and demonstrate how the teacher facilitated the norms, how a joint enterprise was beginning to emerge and how the teacher introduced tools and conventions of the mathematical community.

1. Social Norms

The social norms of the classroom refer to students' normative understandings regarding their participation in mathematics instruction. Social norms such as that students should share solutions, students are to work together on activities and students are to engage in classroom discussions help create an atmosphere of inquiry instruction that contribute to the conditions which make meaningful mathematical learning possible (Cobb, Wood, Yackel, & McNeal, 1992).

In acting as the broker the teacher used various means to help guide the class' mathematical activity. In dealing with the statement I am a grey predator (see Appendix A), two students suggest different methods for finding the solution.

Tyson: Well, uh, because he says he's a predator, that the rational ones can't lie
Teacher: So an important fact was used, rationals can't lie (records statement on board)
Tyson: Because he said he was a predator that part has to be right because otherwise if he said he was rational ... um ... he would have been lying ... I guess ... I'm not exactly sure how to say it
Teacher: Predator part must be true (while recording on board)
Tyson: So since they have to lie on something, they must have lied on the color ... because he is a predator

...

Teacher: A few people have it makes sense? Did somebody have another way to think about it?
Dustin: Well, basically we can take each one and figure out if it would be possible for it to be that one and like cancel them out for each.

...

Teacher: So let's just do one, give me an example and let's do one of these.
Dustin: Like red rational.
Teacher: Ok so could he be a red rational (records on board). So how does that process work?
Dustin: Then you just go back to the fact that the red rational you can say you would say red rational cause they can't lie so since they're ... they can't be ...
Teacher: So could he be a red rational, no (records on board using RR as abbreviation) since red rational can't lie, since he can't lie he would say he's a red rational, he wouldn't say he's a red predator.

Here we see Dustin and the teacher contributing to the social norm of explaining a solution method that is different from the first method. The teacher did not elaborate on how the two methods were alike and thus the students were left to make their own interpretations. By participating in such exchanges the students learned that the teacher valued different approaches to solving a problem as long as they could be understood by the class. Also, the teacher was furthering her pedagogical agenda by writing parts of their solutions that were significant in reaching their conclusions. This made the students suggestions visible to the rest of the class which encouraged the students to reflect on the contributions and offer further comments. By having the offerings explicitly written students were able to clearly see what other students were saying as opposed to having to remember and recall what was said.

The social norms were essential in establishing the classroom community. These social underpinnings were what allowed for meaningful student exchanges and encouraged student negotiations as new concepts were introduced into the class. These kinds of interactions were instrumental in the emergence of the classroom community.

2. Socio-mathematical Norms

The normative aspects of the classroom that are specifically related to mathematics have come to be known as the socio-mathematical norms (Yackel & Cobb, 1996). They refer to what counts as acceptable mathematical activity in the classroom and are jointly constituted between the teacher and the class.

The teacher as a broker was also instrumental in establishing the socio-mathematical norms of the classroom. As the representative of the mathematical community, she was able to question students and influence student conversations so that the proper socio-mathematical norms emerged. The students had discussions, often prompted or sustained by the teacher, that worked to establish the socio-mathematical norms of what constituted a sufficient mathematical solution and when two solutions were equivalent. By sustaining these normative behaviors that are similar to that of the larger mathematical community, she was creating an environment where the students' engagement would begin to resemble that of mathematicians.

The following example documents the students and the teacher negotiating the socio-mathematical norm of what is a mathematically different solution and when a solution is mathematically sufficient. Once the students had reached an agreed upon way to negate an AND statement, the teacher asked them about negating an OR statement. As a class they had discussed a truth table (Figure 1) and used it to demonstrate how to verify the negation of an AND statement. Now the students had to decide how to negate an OR statement and justify their claim. Here is an example where the students were dealing with the Wishy-Washy Dragons problem (see Appendix A).

	p	**q**	**p ∧ q**	$\sim (p \wedge q)$
RR	**T**	**T**	**T**	**F**
RP	**T**	**F**	**F**	**T**
GR	**F**	**T**	**F**	**T**
GP	**F**	**F**	**F**	**T**

Figure 1: Truth Table

Teacher: What about if I negate the OR statement. I'm red or I'm rational ... What's the negation of this going to be?
(The teacher writes question on board)

Tyson: I was thinking since you're switching the ANDs to an OR, you do the same for an OR, you switch to an AND so you would switch to I am grey and I am a predator
Teacher: So could the answer would be grey and predator (writes his response on the board)? Then we had another thought.
Brandon: I was going to say not red and not rational.
Teacher: So another version of this (points to Tyson's response) is not red and not rational (writes it on board). Does that make sense, these two answers?
Dustin: It also make sense if you like, 'cause if you do like, p or q you get (referring to the truth table on the board) true, true, true, false and then if you negate that then you get false, false, false, true and the only one left is the one that has grey and predator, the only one that's true.
Teacher: So if I did p or q, so red or rational I would get true for both, true if either one of them is true, but false if neither is true.
(The teacher adds $p \vee q$ column to the existing table)
Dustin: Right, and if you negate that you get false, false, false, true.
Teacher: So if I want to negate this (writes $p \vee q$) I should get the opposite of this? (points to $p \vee q$ column) is that what you're saying? **Dustin**: Right
Teacher: False, false, false, true?
Dustin: Uh huh, which would make it grey predator.
Teacher: So that grey predator is the only one that's true? Grey and predator.
Dustin: Yeah.
Teacher: Ah ha, thank you. So grey and predator being the only one true. Just another way to write not red and not rational. So this is the other version of DeMorgan's Law. We say DeMorgan's Law for either version, when you negate an AND and you switch to OR or when you negate an OR and you switch to AND with the NOTs.

The students were contributing to the socio-mathematical norm of what constituted a mathematically different solution. Each student was correct and each had his own way of reasoning about the problem. Tyson and Brandon both appeared to be adjusting DeMorgan's Law to fit the negation of an OR statement based on their use of the law for an AND statement while Dustin was able to reason using a truth table. The teacher legitimized all of the solutions by writing them on the board and also made their solutions clear to the class so they could judge the validity of their answers.

As the broker, the teacher was encouraging multiple solutions and helping the students negotiate their similarities and differences. Furthermore, Dustin's response reflected a shift in reasoning regarding the objects in question, or the dragons, to using the tool of a truth table. Thus Dustin's comment served the function of contributing to the constitution of the socio-mathematical norm that a solution correctly based on a classroom tool is mathematically sufficient. Also, by the way the teacher responded to Dustin's solution she indicated that this response was in some way special. Such judgments help support students mathematical learning by helping them become aware of more conceptually advanced forms of mathematical activity while leaving it up to them to decide whether to take up the intellectual challenge (Voigt, 1995).

3. Joint Enterprise

During the semester, as the students and the teacher jointly negotiated the socio-mathematical norm of what constituted a sufficient mathematical justification, they appeared to develop a communal goal of using a deductive argument when justifying a claim. This joint enterprise emerged as they began to create their own deductive arguments as well as requiring them from each other when making a justification. This communal interest was a result of many negotiations within the classroom that were often sustained by the broker through questioning and modeling of deductive arguments.

In the following example we can see the emergence of the joint enterprise that students were to use deductive arguments when justifying their solutions. Here the teacher initiated a discussion about the symmetric property for a bi-conditional statement. The students were discussing conditional statements and their meanings in order to arrive at the conclusion that a bi-conditional statement is symmetric.

Teacher: Does it mater if I write B if and only if A versus A if and only if B?
Michelle: You're saying that both of the implied statements are correct then.
Teacher: Yeah I want to say that both of these (A if B and B if A) are true if I have this (A $\Leftrightarrow$ B), this is the same thing as both of them being true. I'm just finding that only if a little hard all of a sudden.
(The teacher writes A $\Leftrightarrow$ B and B $\Leftrightarrow$ A on the board and asks if they're the same)
Class: Yes.
Jason: Do you want to write a truth table for that? It's probably easier to see
Teacher: Truth tables would that make it easier? Do you have any idea what he's talking about? Hold off for a second on the truth table. Do you have another way to think about it?
Jason: It's the same as the equals from that (points to previous response of A=B)
Teacher: Yeah, it's the same way he was explaining the equals is the same intention as this.

By confirming Jason's response, the teacher was relaying to the class that a bi-conditional statement has similar properties to an equality. In order to point out that they were mathematically different, the teacher continues the discussion.

Teacher: More thoughts?
Brandon: So you're saying A if and only if B is just another way of saying B implies A.
Tyson: And A implies B.
Brandon: But not both ways though.
Several students: Yeah, both ways.
Michelle: Saying that if and only if is the same that both implications are true, you can't have A implies B and B implies A. You can't have a predator that isn't a liar. (the other students seem confused) It's like saying A are predators and B are liars then if a dragon is a predator then it has to be a liar and also if a dragon is a liar then it has to be a predator. If A then B also if B then A. It's a double implication . . . both implications are true.
(Several students agree with her solution)

Here Michelle was able to validate the symmetric property in terms of the dragon statements. In the words of Yackel and Cobb (1996) she gave an explanation as a description of an action on an experientially real mathematical object. In other words, as opposed to a procedural explanation, Michelle used the quantities that A and B represented in order to justify her solution. This can be seen as a contribution to the ongoing negotiation of what constitutes an acceptable explanation in the classroom. Since the original statements were in terms of dragons, the students seem to appreciate the explanation of the symmetric property using statements about dragons giving legitimacy to Michelle's solution.

The norms of participation led to this negotiation of meaning about the symmetric property. The students were reasoning through their explanation and appeared to be tuning their joint enterprise to that of using deductive arguments in order to justify their claims about the symmetric property and conditional statements. It was the broker's role as a facilitator that helped encourage the students' participation.

4. Tools and Conventions

Another important role of the teacher as broker was that she had to introduce the conventions that had been established by the mathematical community. Often these conventions do not have a logical basis and therefore students will have difficulty deriving them or recognizing a need for them. It is the teacher's job to make sure the students use the same symbols as the mathematical community. For example here is the teacher's response when the students had difficulty determining which sets are included in an logical OR statement:

Teacher: Yeah, that's what I was kinda coming to too, we're just going to have to decide and I think in English language, normal speaking, you tend to mean one or the other and not both. So I think more common in English language, especially with the either, is that it's only these two (red solid and grey striped) and not both (red striped). The tricky thing is what mathematicians have come to is that it's more convenient for math to include both ... There's two types of OR. An inclusive OR, it's inclusive because it includes both and then there's another type called an exclusive OR where you do not include both (The teacher writes both definitions on the board) ... For the purposes of this class and for the purposes of math generally if they don't tell you which one, it's an inclusive, it includes both.

Here the teacher had to intervene to clear up what is meant in mathematics by an OR statement as it differs from its everyday use. These situations appeared often in this class. She also had to determine when to introduce the formal mathematical symbols used by the community of mathematicians.

Finally the teacher as broker was responsible for helping students create the formal tools in the classroom community that are used by the community of mathematicians. As an example, while modeling the students' thinking on a problem, the teacher created a grid that resembled the way they were discussing the problem. Noticing its similarity to a truth table, the teacher took the opportunity to introduce a formal truth table which is a tool of the mathematical community. The students were then able to use this tool in working on problems in order to make sense of them, as seen in the example concerning socio-mathematical norms. In order to become a member of a community, whether it is the classroom community or the community of mathematicians, it is important for the individuals to be able to use the tools of that community.

Discussion and Conclusions

In order for a classroom community of practice to emerge students must be actively engaged in the practices of the classroom. The teacher as broker facilitates this engagement by guiding the establishment of the appropriate norms of behavior. She accomplished this by constantly encouraging students to participate in both whole class and small group discussions, making sure students understood and were convinced by solutions given during class and by allowing students to pursue tangents when working on a problem. These kinds of actions helped influence the ways students interacted during class.

As can be seen in the results from the first two weeks of this class, students were beginning to engage during class discussions. They were contributing to what constituted suitable participation in the class while also determining what would be considered appropriate mathematical activity for the class. The teacher helped facilitate these types of engagement by constantly questioning the students and making sure the class was in agreement as new mathematical ideas emerged. As the class progressed through the semester, the class and the broker jointly contributed to the ongoing negotiation of these normative behaviors.

Implications

The results of this study have implications both for practice and for future research. For practice, the study begins to describe what kinds of teacher actions are necessary to foster the emergence of such a community. For example, in order to encourage the norms of the classroom, the teacher effectively questioned the students and allowed wait time to encourage student responses. Uncovering the relationship between particular teacher actions and student responses can help inform future instructors who wish to teach in a classroom centered around activities and student discussions.

Research implications include further description of the framework of community of practice as well as the introduction of the notion of broker for looking at the teacher's role in such a classroom. In looking at classroom environments which foster learning by engaging students in collaborative forms of inquiry, researchers have begun using the concept of community of practice (Goos, 2004). As the term is relatively new to the mathematics education literature, it is important to describe how it emerges in the classroom so that its usefulness can be better understood. In this light it is important to consider the teacher's role in this classroom community. Viewing the teacher as the broker between these two communities gives us the perspective that her actions were done with the intention of influencing the students' activity to resemble that of mathematicians.

Appendix A

The Dragons of Lidd

There are two types of dragons in the Kingdom of Lidd. Rational dragons, being sensible, have determined that devouring farm animals and their owners is, in the long run, not healthy for dragons. Predator dragons, on the other hand, respond to their instincts and refuse to do otherwise, nor do they show any fear of humans.

Dragons in Lidd are also of two different colors related to their veracity. Red rational dragons always tell the truth, and red predator dragons always lie. Grey rational dragons always tell the truth and grey predator dragons always lie.

Because dragons are few in number and are considered an endangered species, the King has decreed that rational dragons shall be protected, and that any knight caught slaying a rational dragon will be dealt with severely.

It would help to know which dragons are rationals and which are predators. It would also help to know a dragon's color. (If one catches a red dragon in a lie, one would know that he is a predator.) Unfortunately, there is an affliction endemic to humans in Lidd: they are colorblind. To them, all dragons look grey.

<u>One Dragon</u>

A knight in full armor and riding a horse approaches a dragon and asks his color and type. Grey rational dragons and red rational dragons always tell the truth; red predator dragons and grey predator dragons always lie.

Dragon: I am a grey predator.

What is he?

<u>Wishy-Washy Dragons</u>

A knight overtakes two dragons in the woods. He pulls his sword and demands to know their color and type.

Dragon A: I am either red or I am rational.

Dragon B: No, I am either red or I am rational. Dragon A is grey.

If the two dragons are different colors, what type of dragons are they? Can you tell which dragon A or B is which type?

Bibliography

[1] Alibert, D. and Thomas, M., Research on Mathematical Proof,Advanced Mathematical Thinking,Tall, D., Kluwer Academic Publishers, Boston: 215 - 230, 1991.

[2] Cobb, P., Conduction Teaching Experiments in Collaboration with Teachers, Handbook of Research Design in Mathematics and Science Education (pp 307-333),Kelly, A. and Lesh, R., editors,Lawrence Erlbaum Associates, Mahweh, NJ, 2000.

[3] Cobb, P.,McClain, K., de Silva Lamburg, T., and Dean, C. Situating Teachers' Instructional Practices in the Instructional Setting of the School and the School District, Educational Researcher,32 (6), pp 12-24, 2003.

[4] Cobb, P. and Hodge, L.L. An Interpretive Scheme for Analyzing the Identities that Students Develop in Mathematics Classrooms, In Press.

[5] Finlow-Bates, K., Lerman, S. and Morgan, C. A Survey of Current Concepts of Proof Held by First Year Mathematics Students, International Proceedings of the 17th Conference of Psychology of Mathematics Education,pp 252-259, 1993.

[6] Goos, M. Learning Mathematics in a Classroom Community of Inquiry, Journal for Research in Undergraduate Mathematics Education,35 (4), pp 258-292, 2004.

[7] Lave, J. and Wenger, E. Situated Learning: Legitimate Peripheral Participation, Cambridge, UK: Cambridge University Pres, 1991.

[8] Moore, R. C. Making the Transition to Formal Proof, Educational Studies in Mathematics,27, pp 249-266, 1994.

[9] Tall, D., The Transition to Advanced Mathematical Thinking: Functions, Limits, Infinity and Proof, Handbook of Research Design in Mathematics Teaching and Learning (pp 495-511),Grouws, D.A., editor,Macmillan: NY, 1992.

[10] Schoenfeld, A.H., Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics, Handbook of Research Design in Mathematics Teaching and Learning (pp 334-370),Grouws, D.A., editor,Macmillan: NY, 1992.

[11] Selden, J. and Selden, A. Unpacking the Logic of Mathematical Statements, Educational Studies in Mathematics,29, pp 123-151, 1995.

[12] Selden, J. and Selden, A. Validations on Proofs Considered as Texts: Can Undergraduates Tell Whether an Argument Proves a Theorem?, Journal for Research in Mathematics Education,34 (1), pp 4-36, 2003.

[13] Steffe, L. and Gale, J. Constructivism in Education,Lawrence Erlbaum: Hillsdale, NJ, 1995.

[14] Voigt, J. Thematic Patterns of Interaction and Sociomathematical Norms, Emergence of Mathematical Meaning: Interaction in Classroom Cultures (pp 163-291),Cobb, P. and Bauserfeld, H., editors,Lawrence Erlbaum: Hillsdale, NJ, 1995.

[15] Weber, K. Student Difficulty in Constructing Proofs: The Need for Strategic KNowledge, Educational Studies in Mathematics,48, pp 101-119, 2001.

[16] Wenger, E. Communities of Practice, New York: Cambridge University Pres, 1998.

[17] Wenger, E. Communities of Practice: Learning, Meaning, and Identity, Cambridge, UK: Cambridge University Pres, 1999.

[18] Yackel, E. and Cobb, P. Sociomathematical Norms, Argumentation, and Autonomy in Mathematics, Journal for Research in Mathematics Education,27, pp 458-477, 1996.

[19] Yackel, E. Explanation, Justification and Argumentation in Mathematics Classrooms, International Proceedings of the 25th Conference of Psychology of Mathematics Education,1, pp 9-24, 2003.