MAT 240

Surface Area, Surface Integrals, Stokes' Theorem and Divergence Theorem Homework

You can complete the following by hand or in MAPLE

1. Set up an integral to calculate the surface area of a sphere of radius 3. Calculate the integral.
2. Find the surface area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.
3. Find the surface area of the function $z = xy$ in between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

If we have a function, $f(x, y, z)$, which gives the charge density (charge per unit area) over a surface S, then the surface integral would give us the total charge on the surface of S.

4. Calculate the total charge on a cone whose base is the disk $x^2 + y^2 = 4$ in the xy plane with a height of 4cm and a charge density of $f(x, y, z) = x + y + z$ C/cm^2.
5. Find the surface integral of the function $f(x, y, z) = xyz$ over the surface given by $x + y + z = 1$ in the first octant.
6. Calculate the surface integral of $\vec{F} =< y, z, x >$ over the surface given by $3x - 4y + z = 1$, $0 \le x, y \le 1$, with an upward-pointing normal.
7. Find the flux of the velocity vector field $\vec{F} =< 0, yz, z^2 >$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1,\ z \ge 0$, by the planes $x = 0$ and $x = 1$.
8. Calculate the line integral of the path given by moving in a semi-circle from (1,0,0) to (0,1,0), a straight line from (0,1,0) to (0,0,1) and a straight line from (0,0,1) back to (1,0,0) through the vector field $\vec{F} =< y, z, x >$.
    a. Using Stoke's Theorem
    b. Using the LineInt command
9. Use the Divergence Theorem to calculate the surface integral $\iint_S \vec{F} \cdot dS$ with $\vec{F} = \langle x, y, z \rangle$ and S is a sphere centered at the origin with a radius of 2. Confirm your answer by computing the surface integral.
10. Use the divergence theorem to compute the flux of $\vec{F} = \langle 3x, z, y + 2x \rangle$ through the boundary of the region contained between a cylinder centered at the origin with a radius of 3, the plane $z = x + 1$ and the xy plane (be sure to look at your solid!).