

GUIDELINES FOR PREPARING FOR SCC's MAT 230 GATEWAY EXAM

The Gateway Exam for MAT 230 contains 10 problems from the following list of topics.

- I. Use an appropriate integration technique (u -substitution, by parts, trigonometric substitution, algebraic simplification) to evaluate a definite or indefinite integral.
- II. Interpret a definite integral $\int_a^b f(x) dx$ (using language provided by the context) or represent a described quantity in terms of an integral.
- III. Determine convergence and divergence of improper integrals, including use of the Comparison Test.
- IV. Analyze and/or graph a function given the graph of its derivative *and* an initial value.
- V. Write a definite integral whose value gives the solution of an applied problem involving area, volume, work, total force (due to fluid pressure), or arclength.
- VI. Find the average value of f over $[a, b]$.
- VII. Find a Riemann sum estimate for $\int_a^b f(x) dx$, where f is a function represented by a table of data, and decide if the sum is an over-estimate or an under-estimate (or neither).

The Gateway Exam is a pencil & paper proficiency exam covering major topics in the MAT 230 Calculus and Analytic Geometry course at Scottsdale Community College. Students must show all appropriate work and give exact answers (with appropriate units, when needed). **While calculators may NOT be used on the Gateway, you need simplify numerical expressions ONLY when asked to do so. No other resource (e.g. integral tables, notes) may be used while taking the exam. You will have 50 minutes to take the exam.**

MAT 230 GATEWAY PRACTICE EXAM

In Problems 1-4, evaluate the integrals.

1. $\int \frac{(x-3)^2}{x^2} dx$

2. $\int_b^4 \sqrt{2x+1} dx$

3. $\int \frac{1}{x^2 + 4x + 5} dx$

4. $\int \frac{\cos x}{\sqrt{1 + \sin x}} dx$

5. At 9 A.M., a pump was turned on in order to add water to a holding tank. If $r(t)$ gallons per minute is the *rate* at which water is being pumped into the tank, where t is the time (in minutes) since the pump was first turned on), **find an integral** that represents the number of gallons of water that was pumped into the tank for the hour from 10:00 to 11:00 A.M.

6. a. Using the limit definition of an improper integral, determine whether $\int_1^{\infty} \frac{1}{x} dx$ converges or diverges.

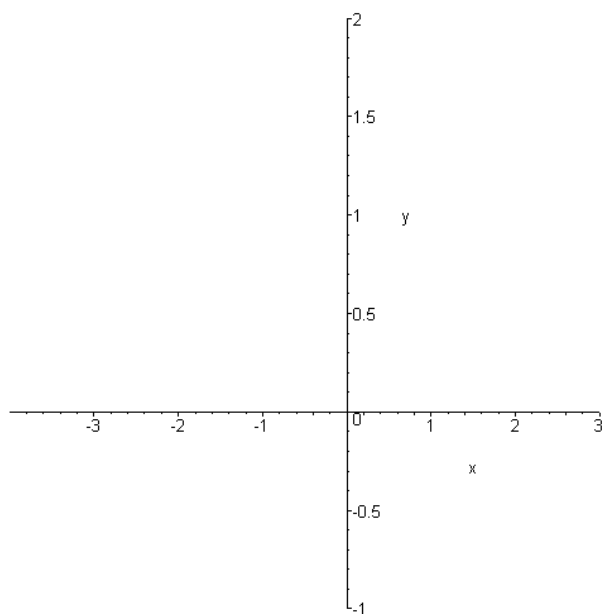
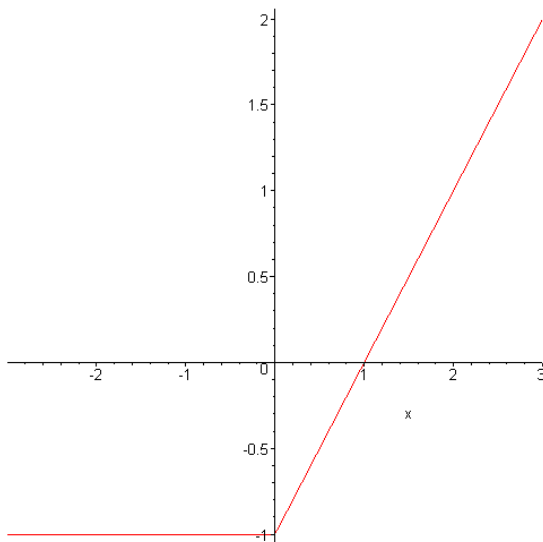
b. From the following three integrals, choose one whose convergence or divergence can be proved by comparison to the integral in part (a), and give the proof.

i. $\int_1^{\infty} \frac{2 + \sin(x)}{x} dx$

ii. $\int_1^{\infty} \frac{1}{x + \sin^2(x)} dx$

iii. $\int_1^{\infty} \frac{1}{x + \sin(x)} dx$

7. For the function f whose graph is given, graph the *antiderivative* that satisfies the initial condition: $F(0) = 0$. **Note:** To receive full credit the antiderivative you sketch must not only have the correct *shape*, it must also take on its correct *values*; use the scale.



8. Consider the region bounded by the first quadrant, $y = ax^2$, and the line $y=c$ (where a and c are positive constants). Revolving this region about the line $x = -2$ produces a solid. **Set up and evaluate** an integral whose value represents the volume of that solid.

9. When a projectile is fired vertically upward from the ground at an initial velocity of 80 ft/sec, its height h (in feet) after t seconds is given by

$$h(t) = 80t - 16t^2.$$

Find the *average height* of the projectile during its 5-second flight.

10. Find the **trapezoidal sum** estimate for $\int_2^{10} f(x) dx$, where f is the increasing function represented by

x	2	4	6	8	10
$f(x)$	1	5	8	10	11

If f has no inflection points, is the trapezoidal sum an over- or an underestimate (or neither)?

The next two problems are variations on some of the previous problem regarding work. There will only be 10 problems on the gateway exam.

Water is to be raised from a well 55 feet deep by means of a bucket attached to a rope that weighs 7 pounds/foot. When the bucket is full of water it weighs 30 pounds. Find the work done in raising the bucket to the top.

A coffee mug in the shape of a cylinder that has a 6 inch radius and is one foot tall and has a 2 ft straw placed in it (so that only 1 ft of the straw is protruding out of the top of the mug). Find the work required to drink **half** of the coffee out through the straw if the mug **is full** of coffee weighing 70 pounds per cubic ft.