Objectives for Function Composition Activity

- Compose two functions give algebraically, by a table, by a graph, in words
- Decompose a function into two (or more) functions
- Compose more than two functions
- Determine the domain of the composition of two functions
Function Composition

1. Given \( f(x) = 3x + 4, \ g(x) = x^2 + 1, \) and \( h(x) = \frac{2}{x-5} \) find:

a. \( f(g(0)) = \)

b. \( g(f(0)) = \)

c. \( f(g(2)) = \)

d. \( g(f(1)) = \)

e. \( f(g(x)) = \)

f. \( g(f(x)) = \)

g. \( h(f(x)) = \)

h. \( f(h(x)) = \)
2. Given that \( h(x) = f(g(x)) \), fill out the table of values for \( h(x) \).

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<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
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3. Given that \( h(x) = f(g(x)) \), fill in the missing values

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4. Use the graphs below to evaluate:
   a) \( f(g(6)) = \) __________________________
   b) \( g(f(2)) = \) __________________________
   c) \( g(f(0)) = \) __________________________

5. Use the Graph of \( f \) and the table for \( g \) to evaluate the following:
   a) \( f(g(4)) = \) ______________
   b) \( g(f(2)) = \) ______________
   c) \( f(g(2)) = \) ______________
6. Let \( f(x) = x^2 + 1 \) and \( g(x) = 2x + 3 \).

a. \( f(7) = \) __________  
b. \( g(3) = \) __________  
c. \( f(g(3)) = \) __________

d. \( f(g(x)) = \) ___________________  
e. \( g(f(x)) = \) ___________________

7. Use the words *input* and *output*, as appropriate, to fill in the missing blanks:

The function \( f(g(t)) \) uses the ______________ of the function \( g \) as the ______________ to the function \( f \). The function \( g(f(t)) \) uses the ______________ of the function \( f \) as the ______________ to the function \( g \).

8. Let \( u(x) = p(q(x)) \) and \( v(x) = q(p(x)) \) where \( p(x) = 3x - 4 \) and \( q(x) = x^2 + 5 \).

a. Calculate \( u(4) \) and \( v(4) \). Are they the same?

\[ u(4) = \] _______  
\[ v(4) = \] _______

b. Find formulas for \( u(x) \) and \( v(x) \) in terms of \( x \). What can you conclude about the order of functions in doing a composition?

\[ u(x) = \] ___________________

\[ v(x) = \] ___________________
9. Let \( f(x) = x^2 + 3 \) and \( g(x) = 2x + 1 \).

a. \( f(7) = \)

b. \( g(3) = \)

c. \( f(g(3)) = \)

d. \( f(f(3)) = \)

e. \( f(g(x)) = \)

f. \( g(f(x)) = \)

g. \( g(g(x)) = \)
Decomposition of Functions

Just as we can compose two functions to create a new function, we can decompose a function into two separate functions, one being the input of the other.

**Warm-up:** Use the words *input* and *output*, as appropriate, to fill in the missing blanks:

The function \( f(g(t)) \) uses the ______ of the function \( g \) as the ________ to the function \( f \). The function \( g(f(t)) \) uses the ______ of the function \( f \) as the ________ to the function \( g \).

1. Let \( g(x) = \frac{1}{x + 1} \). Decompose \( g \) into functions, \( f \) and \( h \), such that \( g(x) = f(h(x)) \). [Do not use \( f(x) = x \) or \( g(x) = x \)]

   \[ h(x) = \quad \]  

   \[ f(x) = \quad \]  

2. Consider the composite function \( w(x) = \sqrt{1 + x^2} \).

   Find two functions (\( f \) and \( g \)) such that \( w(x) = f(g(x)) \). [Do not use \( f(x) = x \) or \( g(x) = x \)]

   \[ g(x) = \quad \]  

   \[ f(x) = \quad \]
3. Consider the composite function \( w(x) = \sqrt{1 + x^2} \).

Find three functions \( f, g, \) and \( h \) such that \( w(x) = f(g(h(x))) \). [Do not use \( f(x) = x, g(x) = x, \) or \( h(x) = x \)]

\[ h(x) = \text{________________________} \]

\[ g(x) = \text{________________________} \]

\[ f(x) = \text{________________________} \]

4. Now consider the composite function \( f(x) = 3(x - 1)^2 + 5 \). Decompose \( f \) into three functions, \( u, v, \) and \( w \), such that \( f(x) = u(v(w(x))) \). [Do not use \( u(x) = x, v(x) = x, \) or \( w(x) = x \)]

\[ w(x) = \text{____________} \]

\[ v(x) = \text{____________} \]

\[ u(x) = \text{____________} \]

Can you see a way to decompose \( f \) into four functions? Demonstrate how to do it:
Domain of a Composition

When finding the domain of a composition we have to take into consideration the domains of the ‘inside’ and ‘outside’ functions as well as the domain of our composition. For example:

Given \( f(x) = x^2 \) and \( g(x) = \sqrt{x} \), then our two composed functions are
\[
\begin{align*}
    h(x) &= f(g(x)) = (\sqrt{x})^2 = x \\
    k(x) &= g(f(x)) = \sqrt{x^2} = x
\end{align*}
\]
They both simplify down to \( x \), but are not the exact same function as they have different domains. The domain of \( h(x) \) is \([0, \infty)\) since we cannot input negative numbers into \( \sqrt{x} \) which is the inside function of the composition. On the other hand, the domain of \( k(x) \) is \((-\infty, \infty)\) because our input goes into \( x^2 \) first which has a domain of all real numbers.

Example 2

Given \( f(x) = x^2 - 4 \) and \( g(x) = \sqrt{x} \). For our composition \( h(x) = g(f(x)) = \sqrt{x^2 - 4} \) we have an inside function that has a domain of all real numbers, but an outside function that has a domain of \([0, \infty)\) so we must make sure that we only get non-negative numbers from our inside function. Now \( f(x) = x^2 - 4 \) is negatives for inputs between -2 and 2, so we must exclude those from the domain of \( h(x) \). Thus we get \((-\infty, -2) \cup (2, \infty)\) for our domain.

Find the following compositions, \( f(g(x)) \) and \( g(f(x)) \) and their domains.

1. \( f(x) = x^2, \quad g(x) = \frac{1}{x - 4} \)

2. \( f(x) = \sqrt{x - 4}, \quad g(x) = \frac{1}{x} \)